

STANDARD  
& POOR'S

# INDEX MATHEMATICS

METHODOLOGY

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# Table of Contents

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Introduction	3
<b>Different Varieties of Indices</b>	<b>3</b>
<b>The Index Divisor</b>	<b>3</b>
Capitalization Weighted Indices	5
<b>Definition</b>	<b>5</b>
<b>Adjustments to Share Counts</b>	<b>6</b>
<b>Divisor Adjustments</b>	<b>6</b>
<b>Necessary Divisor Adjustments</b>	<b>8</b>
<b>Capped Indices</b>	<b>10</b>
Equal Weighted Indices	11
<b>Definition</b>	<b>11</b>
<b>Corporate Actions and Index Adjustments</b>	<b>12</b>
<b>Index Actions</b>	<b>13</b>
<b>Corporate Actions</b>	<b>13</b>
Modified Market Capitalization Weighted Indices	14
<b>Definition</b>	<b>14</b>
<b>Corporate Actions and Index Adjustments</b>	<b>16</b>
<b>Index Actions</b>	<b>16</b>
<b>Corporate Actions</b>	<b>16</b>
Attribute Weighted Indices	17
<b>Corporate Actions</b>	<b>19</b>
Leveraged and Inverse Indices	21
<b>Leveraged Indices</b>	<b>21</b>
<b>Inverse Indices</b>	<b>22</b>
Total Return Calculations	24
Index Fundamentals	25

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Fund Strategy Indices	26
<b>Total Return and Synthetic Price Indices</b>	<b>27</b>
<b>Index and Corporate Actions</b>	<b>28</b>
S&P Contact Information	29
<b>Index Management</b>	<b>29</b>
<b>Media Relations</b>	<b>29</b>
<b>Index Operations &amp; Business Development</b>	<b>29</b>
Disclaimer	30

# Introduction

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This document covers the mathematics of index calculations. To understand and successfully use indices for investment analyses, it is important to know how they are calculated and how adjustments are made when constituents change or when different kinds of corporate actions occur. While actual index calculations are done almost entirely by computer, utilizing a wide range of programs, algorithms and routines, the underlying math is fairly straightforward. This document assumes some acquaintance with mathematical notation and simple operations, and little more. At the same time, the calculations are presented principally as equations, with a few examples or tables of results. The equations – which have largely been excluded from the individual index methodologies – are the most efficient way to present the information.

## **Different Varieties of Indices**

A majority of Standard & Poor's equity indices are market cap weighted and float-adjusted, where each stock's weight in the index is proportional to its float-adjusted market value. The first section covers these indices. A second group of indices are equal weighted, where each stock is weighted equally in the index. These are discussed in the second section. A third group of indices are weighted by other factors, such as maximum weight restrictions or certain attributes used to choose the stocks. These are discussed in sections three and four. A fourth group of indices are leverage and inverse indices, which return positive or negative multiples of their underlying indices. These are discussed in section five.

There are also calculations that operate on an index as a whole rather than on the individual stocks. These include calculations of total returns and index fundamentals. These are covered in sections six and seven below.

Fund strategy indices, another group of indices, include portfolios of funds. These indices are covered in section eight.

## **The Index Divisor**

Throughout all the calculations there is one concept that is crucially important to understanding how indices are calculated – the index divisor.

The simplest capitalization weighted index can be thought of as a portfolio consisting of all available shares of the stocks in the index. While one might track this portfolio's value in dollar terms, it would probably be an unwieldy number – for example, the S&P 500 market value is roughly \$11.8 trillion. Rather than deal with ten or more digits, the

figure is scaled to a more easily handled number, currently around 1250. Dividing the portfolio market value by a factor, usually called the divisor, does the scaling.

An index is not exactly the same as a portfolio. For instance, when a stock is added to or deleted from an index, the index level should not jump up or drop down; while a portfolio's value would usually change as stocks are swapped in and out. To assure that the index's value, or level, does not change when stocks are added or deleted, the divisor is adjusted to offset the change in market value of the index. Thus, the divisor plays a critical role in the index's ability to provide a continuous measure of market valuation when faced with changes to the stocks included in the index. In a similar manner, some corporate actions that cause changes in the market value of the stocks in an index should not be reflected in the index level. Adjustments are made to the divisor to eliminate the impact of these corporate actions.

# Capitalization Weighted Indices

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Most of Standard & Poor's indices, indeed most widely quoted stock indices, are capitalization-weighted indices. Sometimes these are called value-weighted or market cap weighted instead of capitalization weighted. Examples include the S&P 500, the S&P Global 1200 and the S&P/Citigroup indices. Examples from other index providers (where some of the details may vary slightly from those described here) include MSCI's indices, FTSE's indices and Russell's indices. While Dow Jones does offer market cap weighted indices, the well-known Dow Jones Industrial Average is not cap-weighted.

In the discussion below most of the examples refer to the S&P 500 but apply equally to a long list of S&P cap-weighted indices.

## Definition

The formula to calculate the S&P 500 is:

$$Index\ Level = \frac{\sum_i P_i * Q_i}{Divisor} \quad (1)$$

The numerator on the right hand side is the price of each stock in the index multiplied by the number of shares used in the index calculation. This is summed across all the stocks in the index. The denominator is the divisor. If the sum in the numerator is \$11.8 trillion and the divisor is \$9.4 billion, the index level would be 1250, close to the current levels of the S&P 500.

This index formula is sometimes called a “base-weighted aggregative” method.<sup>1</sup> The formula is created by a modification of a *LasPeyres* index, which uses base period quantities (share counts) to calculate the price change. A *LasPeyres* index would be:

$$Index = \frac{\sum_i P_{i,t} * Q_{i,o}}{\sum_i P_{i,0} * Q_{i,o}} \quad (2)$$

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<sup>1</sup> This term is used in one of the earlier and more complete descriptions of S&P index calculations in Alfred Cowles, *Common Stock Indices*, Principia Press for the Cowles Commission of Research in Economics, 1939. The book refers to the “Standard Statistics Company Formula;” S&P was formed by the merger of Standard Statistics Corporation and Poor's Publishing in 1941.

In the modification to (2), the quantity measure in the numerator,  $Q_0$ , is replaced by  $Q_1$ , so the numerator becomes a measure of the current market value, and the product in the denominator is replaced by the divisor which both represents the initial market value and sets the base value for the index. The result of these modifications is equation (1) above.

### Adjustments to Share Counts

Standard & Poor's market cap-weighted indices are float adjusted – the number of shares outstanding is reduced to exclude closely held shares from the index calculation because such shares are not available to investors. S&P's rules for float adjustment are described in more detail in *Standard & Poor's Float Adjustment Methodology* or in some of the individual index methodology documents. As discussed there, for each stock S&P calculates an Investable Weight Factor (IWF) which is the percentage of total shares outstanding that are included in the index calculation. When the index is calculated using equation (1), the variable  $Q_i$  is replaced by the product of outstanding shares and the IWF:

$$Q_i = IWF_i * Total\ Shares_i \quad (3a)$$

At times there are other adjustments made to the share count to reflect foreign ownership restrictions or to adjust the weight of a stock in an index. These are combined into a single multiplier in place of the IWF in equation (3a). In combining restrictions it is important to avoid unwanted double counting. Let FA represent the fraction of shares eliminated due to float adjustment, FR represent the fraction of shares excluded for foreign ownership restrictions and IS represent the fraction of total shares to be excluded based on the combination of FA and FR.

$$\text{If } FA > FR \text{ then } IS = 1 - FA, \quad (4a)$$

$$\text{If } FA < FR \text{ then } IS = 1 - FR \quad (4b)$$

and equation (3a) can be written as:

$$Q_i = IS_i * Total\ Shares_i \quad (3b)$$

Note that any time the share count or the IWF is changed, it will be necessary to adjust the index divisor to keep the level of the index unchanged.

### Divisor Adjustments

The key to index maintenance is the adjustment of the divisor. Index maintenance – reflecting changes in shares outstanding, capital actions, addition or deletion of stocks to the index – should not change the level of the index. If the S&P 500 closes at 1250 and one stock is replaced by another, after the market close, the index should open at 1250 the next morning if all of the opening prices are the same as the previous day's closing prices. This is accomplished with an adjustment to the divisor.

Any change to the stocks in the index that alters the total market value of the index while holding stock prices constant will require a divisor adjustment. This section explains how the divisor adjustment is made given the change in total market value. The next section discusses what index changes and corporate actions lead to changes in total market value and the divisor.

Equation (1) is expanded to show the stock being removed, stock  $r$ , separately from the stocks that will remain in the index:

$$Index\ Level_{t-1} = \frac{(\sum_i P_i * Q_i) + P_r Q_r}{Divisor_{t-1}} \quad (5)$$

Note that the index level and the divisor are now labeled for the time period  $t-1$  and, to simplify this example, that we are ignoring any possible IWF and adjustments to share counts. After stock  $r$  is replaced with stock  $s$ , the equation will read:

$$Index\ Level_t = \frac{(\sum_i P_i * Q_i) + P_s Q_s}{Divisor_t} \quad (6)$$

In equations (5) and (6)  $t-1$  is the moment right before company  $r$  is removed from and  $s$  is added to the index;  $t$  is the moment right after the event. By design,  $Index\ Level_{t-1}$  is equal to  $Index\ Level_t$ . Combining (5) and (6) and re-arranging, the adjustment to the Divisor can be determined from the index market value before and after the change:

$$\frac{(\sum_i P_i * Q_i) + P_r Q_r}{Divisor_{t-1}} = Index\ Level = \frac{(\sum_i P_i * Q_i) + P_s Q_s}{Divisor_t} \quad (7)$$

Let the numerator of the left hand fraction be called  $MV_{t-1}$ , for the index market value at  $(t-1)$ , and the numerator of the right hand fraction be called  $MV_t$ , for the index market value at time  $t$ . Now,  $MV_{t-1}$ ,  $MV_t$  and  $Divisor_{t-1}$  are all known quantities. Given these, it is easy to determine the new divisor that will keep the index level constant when stock  $r$  is replaced by stock  $s$ :

$$Divisor_t = (Divisor_{t-1}) * \frac{MV_t}{MV_{t-1}} \quad (8)$$

As discussed below, various index adjustments result in changes to the index market value. When these adjustments occur, the divisor is adjusted as shown in equation (8).

In some implementations, including the computer programs used in S&P's index calculations, the divisor adjustment is calculated in a slightly different, but equivalent, format where the divisor change is calculated by addition rather than multiplication. This alternative format is defined here. Rearranging equation (1) and using the term MV (market value) to replace the summation gives:

$$Divisor = \frac{MV}{IndexLevel} \quad (9)$$

When stocks are added to or deleted from an index there is an increase or decrease in the index's market value. This increase or decrease is the market value of the stocks being added less the market value of those stocks deleted; define CMV as the Change in Market Value. Recalling that the index level does not change, the new divisor is defined as:

$$Divisor_{New} = \frac{MV + CMV}{IndexLevel} \quad (10)$$

or

$$Divisor_{New} = \frac{MV}{IndexLevel} + \frac{CMV}{IndexLevel} \quad (11)$$

However, the first term on the right hand side is simply the Divisor value before the addition or deletion of the stocks. This yields:

$$Divisor_{New} = Divisor_{Old} + \frac{CMV}{IndexLevel} \quad (12)$$

Note that this form is more versatile for computer implementations. With this additive form, the second term ( $CMV/IndexLevel$ ) can be calculated for each stock or other adjustment independently and then all the adjustments can be combined into one change to the Divisor.

### Necessary Divisor Adjustments

Divisor adjustments are made "after the close" meaning that after the close of trading the closing prices are used to calculate the new divisor based on whatever changes are being made. It is, then, possible to provide two complete descriptions of the index – one as it existed at the close of trading and one as it will exist at the next opening of trading. If the same stock prices are used to calculate the index level for these two descriptions, the index levels are the same.

With prices constant, any change that changes the total market value included in the index will require a divisor change. For cataloging changes, it is useful to separate changes caused by the management of the index from those stemming from corporate actions of the constituent companies. Among those changes driven by index management are adding or deleting companies, adjusting share counts and changes to IWFs and other factors affecting share counts or stock prices.

**Index Management Related Changes:** When a company is added to or deleted from the index, the net change in the market value of the index is calculated and this is used to calculate the new divisor. The market values of stocks being added or deleted are based on the prices, shares outstanding, IWFs and any other share count adjustments. Specifically, if a company being added has a total market cap of US\$ 1 billion, an IWF of 85% and, therefore, a float adjusted market cap of US\$ 850 million, the market value for the added company used is US\$ 850 million. The calculations would be based on either equation (8) or equation (12) above.

For most S&P indices, there are a few dates during the year when IWFs and share counts are updated. (Typically small changes in shares outstanding are reflected in indices once a quarter to avoid excessive changes to an index.) The revisions to the divisor resulting from these are calculated and a new divisor is determined. Equation (12) shows how the impact of a series of share count changes can be combined to determine the new divisor.

**Corporate Action Related Changes:** There are a large range of different corporate actions ranging from routine share issuances or buy backs to unusual events like spin-offs or mergers. These are listed on the table below with notes about the necessary changes and whether the divisor is adjusted.

<b>Corporate Action</b>	<b>Comments</b>	<b>Divisor Adjustment</b>
Company added/deleted	Net change in market value determines the divisor adjustment.	Yes
Change in shares outstanding	Any combination of secondary issuance, share repurchase or buy back – share counts revised to reflect change.	Yes
Stock split	Share count revised to reflect new count. Divisor adjustment is not required since the share count and price changes are offsetting.	No
Spin-off	If the spun-off company is not being added to the index, the divisor adjustment reflects the decline in index market value (i.e., the value of the spun-off unit).	Yes
Spin-off	Spun-off company added to the index, no company removed from the index.	No
Spin-off	Spun-off company added to the index, another company removed to keep number of names fixed. Divisor adjustment reflects deletion.	Yes
Change in IWF	Increasing (decreasing) the IWF increases (decreases) the total market value of the index. The divisor change reflects the change in market value caused by the change to an IWF.	Yes
Special Dividend	When a company pays a special dividend the share price is assumed to drop by the amount of the dividend; the divisor adjustment reflects this drop in index market value.	Yes
Rights offering	Each shareholder receives the right to buy a proportional number of additional shares at a set (often discounted) price. The calculation assumes that the offering is fully subscribed. Divisor adjustment reflects increase in market cap measured as the shares issued multiplied by the price paid.	Yes

With corporate actions where cash or other corporate assets are distributed to shareholders, the price of the stock will gap down on the ex-dividend day (the first day when a new shareholder is not eligible to receive the distribution.) The effect of the divisor adjustment is to prevent this price drop from causing a corresponding drop in the index.

### **Capped Indices**

At times it is desirable to set a maximum weight for some stocks in an index. In some markets regulations restrict the weight of the largest stock or group of stocks to be less than a certain percentage of a portfolio. This is done by a further adjustment to the share count, beyond the investable weight factor. Since the total weight of all stocks in the index will add up to 100%, reducing the weight of one stock will increase the weight of the others. It is possible that when the largest stock's weight is brought down below some limit, the weight of the next largest – or several next largest – stocks will exceed the limit. Therefore, the process must be iterative. Weights will change over time as stock prices move even if share counts remain constant. If a capped stock enjoys a price run-up, it may exceed the cap. In most cases buffer zones are used – e.g., if the maximum allowable weight is 10% the stock's shares are adjusted downward until its weight is 9% leaving a one-percentage point buffer before another adjustment is necessary.

# Equal Weighted Indices

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## Definition

An equal weighted index is one where every stock has the same weight in the index, and a portfolio that tracks the index will invest an equal dollar amount in each security. As stock prices move, the weights will shift and exact equality will be lost. Therefore, an equal weighted index must be rebalanced from time to time to re-establish the proper weighting. (In contrast, a cap-weighted index requires no rebalancing as long as there aren't any changes to share counts, IWFs, returns of capital, or stocks added or deleted.)

The overall approach to calculate equal weighted indices is the same as in the cap-weighted indices; however, the constituents' market values are re-defined to be values that will achieve equal weighting at each rebalancing. Recall two basic formulae:

$$\text{Index Level} = \frac{\text{Index Market Value}}{\text{Divisor}} \quad (13)$$

and

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * \text{IWF}_i \quad (14)$$

To calculate an equal weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has an equal weight in the index at each rebalancing date. In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish equal weighting.

$$\text{Adjusted Stock Market Value}_i = P_i * \text{Shares}_i * \text{IWF}_i * \text{ExRate}_i * \text{AWF}_i \quad (15)$$

where  $\text{AWF}_i$  is the adjustment factor of stock  $i$  assigned at each index rebalancing date,  $t$ , which makes all index constituents modified market capitalization equal (and, therefore, equal weight), while maintaining the total market value of the overall index. The  $\text{AWF}$  for each index constituent,  $i$ , at rebalancing date,  $t$ , is calculated by:

$$\text{AWF}_{i,t} = \frac{Z}{N * \text{FloatAdjustedMarketValue}_{i,t}} \quad (16)$$

where  $N$  is the number of stocks in the index and  $Z$  is an index specific constant set for the purpose of deriving the  $AWF$  and, therefore, each stock's share count used in the index calculation (often referred to as modified index shares).

The index divisor is defined based on the index level and market value from equation (13). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

So:

$$(Divisor)_{after\ rebalancing} = \frac{(Index\ Market\ Value)_{after\ rebalancing}}{(Index\ Value)_{before\ rebalancing}} \quad (16a)$$

where,

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i * ExRate_i * AWF_i \quad (16b)$$

### Corporate Actions and Index Adjustments

The tables on the following page show the necessary adjustments to the index and the divisor for managing an equal weighted index. One key issue is how to handle events when one stock is replaced by another. Given that stock prices move all the time, the index is only truly equally weighted at the rebalancing. Therefore, when stocks are added or deleted either the new stock must assume the actual weight of the old stock or the entire index must be rebalanced. Since index rebalancing generates trading costs for tracker funds, the design decision is usually made to have a new stock assume the weight of the stock being dropped until the next rebalancing.

## Index Actions

<b>S&amp;P parent index action</b>	<b>Adjustment made to the equal weight index</b>	<b>Divisor adjustment for the S&amp;P EWI</b>
Constituent change – even number of adds and drops	The company entering the index goes in at the weight of the company coming out. This weight is used to compute the adjusted weight factor of the added stock, using Equation 15. If a company is being removed at a price of 0.00, the replacement goes in at the weight of the deleted company at the close on the day before the effective date. If more than one company is being replaced in the index on a single date, the replacements would be in the order stated in the press release for the parent index change.	None
Constituent change – deletion only	The weights of all stocks in the index will change, due to the absolute change in the number of index constituents. Relative weights will stay the same.	Yes
Share changes between quarterly share adjustments	None. The adjustment factor is changed to keep the index weight the same.	None
Quarterly share changes	There is no direct adjustment.	None

## Corporate Actions

<b>S&amp;P parent index action</b>	<b>Adjustment made to the equal weight index</b>	<b>Divisor adjustment for the S&amp;P EWI</b>
Spin-off	The price is adjusted to the Price of the Parent Company minus (the Price of Spin-off company/Share Exchange Ratio). The adjustment factor changes according to Equation 15, to maintain the weight to be the same as the company had before the spin-off.	None
Rights Offering	The price is adjusted to the Price of the Parent Company minus (the Price of Rights Offering/Rights Ratio). The adjustment factor changes according to Equation 15, to maintain the weight to be the same as the company had before the rights offering.	None
Stock Split	Shares are multiplied by and the price is divided by the split factor.	None
Share Issuance or Share Repurchase	None.	None
Special Dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	A divisor adjustment is made to ensure the index level remains the same.

# Modified Market Capitalization Weighted Indices

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## Definition

A modified market cap weighted index is one where index constituents have a user-defined weight in the index. This methodology is typically used for indices where some constituents are confined to a maximum weight (sometimes called capped indices), and the excess weight is distributed proportionately among the remaining index constituents. As stock prices move, the weights will shift and the modified weights will change. Therefore, as in the case of an equal-weighted index, a modified market cap weighted index must be rebalanced from time to time to re-establish the proper weighting.

The overall approach to calculate modified market cap weighted indices is the same as in the cap-weighted indices; however, the constituents' market values are re-defined to be values that will achieve the user-defined weighting at each rebalancing. Recall two basic formulae:

$$\text{Index Level} = \frac{\text{Index Market Value}}{\text{Divisor}} \quad (17)$$

and

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * \text{IWF}_i \quad (18)$$

To calculate a modified market cap weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has the appropriate user-defined weight in the index at each rebalancing date.

In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish the appropriate weighting.

$$\text{Adjusted Stock Market Value}_i = P_i * \text{Shares}_i * \text{IWF}_i * \text{ExRate}_i * \text{AWF}_i \quad (19)$$

where  $\text{AWF}_i$  is the adjustment factor of stock  $i$  assigned at each index rebalancing date,  $t$ , which adjusts the market capitalization for all index constituents to achieve the user-defined weight, while maintaining the total market value of the overall index.

The *AWF* for each index constituent, *i*, on rebalancing date, *t*, is calculated by:

$$AWF_{i,t} = \frac{Z}{FloatAdjustedMarketValue_{i,t}} * W_{i,t} \quad (20)$$

where *Z* is an index specific constant set for the purpose of deriving the *AWF* and, therefore, each stock's share count used in the index calculation (often referred to as modified index shares). *W<sub>i,t</sub>* is the user-defined weight of stock *i* on rebalancing date *t*.

The index divisor is defined based on the index level and market value from equation (17). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

So:

$$(Divisor)_{after\ rebalancing} = \frac{(Index\ Market\ Value)_{after\ rebalancing}}{(Index\ Value)_{before\ rebalancing}} \quad (20a)$$

where,

$$Index\ Market\ Value = \sum_i P_i * Shares_i * IWF_i * ExRate_i * AWF_i \quad (21b)$$

## Corporate Actions and Index Adjustments

The tables below shows the necessary adjustments to the index and the divisor for managing a modified market cap weighted index.

### Index Actions

S&P parent index action	Adjustment made to the modified market cap weighted index	Divisor adjustment for the index?
Constituent change	The company entering the index goes in at the weight of the company coming out.	None
Delisting, acquisition or any other corporate action resulting in a constituent deletion.	The stock is dropped from the Index	Yes
Share changes between quarterly share adjustments	None. The adjustment factor is changed to keep the index weight the same.	None
Quarterly share changes	There is no direct adjustment.	None

### Corporate Actions

S&P parent index action	Adjustment made to the modified market cap weighted index	Divisor adjustment for the index?
Spin-off	The price is adjusted to the Price of the Parent Company minus (the Price of Spin-off company/Share Exchange Ratio). The adjustment factor changes according to Equation 20, to maintain the weight to be the same as the company had before the spin-off.	None
Rights Offering	The price is adjusted to the Price of the Parent Company minus (the Price of Rights Offering/Rights Ratio). The adjustment factor changes according to Equation 20, to maintain the weight to be the same as the company had before the rights offering.	None
Stock Split	Shares are multiplied by and the price is divided by the split factor.	None
Share Issuance or Share Repurchase	None.	None
Special Dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	A divisor adjustment is made to ensure the index level remains the same.
Merger or acquisition	If the surviving company is already an index member, it is retained in the index. If the surviving company does not meet index criteria, it is removed.	Yes, if there is a removal.

# Attribute Weighted Indices

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In recent years various new approaches to weighting stocks in indices have appeared to supplement cap weighting and price weighting.<sup>2</sup> S&P's Pure Style Indices, introduced in 2005, are attribute weighted – a stock's weight depends on the measures of its growth or value attributes, the same measure that was used to select stocks for the index. The discussion here covers how these indices are calculated; the selection of stocks is covered in the S&P U.S. Style Indices methodology document.

There are both Pure Growth Style and Pure Value Style indices. Under the selection process, each stock has a growth score and a value score. These scores are used to identify pure growth stocks and pure value stocks. (A stock cannot be both pure growth and pure value; it can be neither pure growth nor pure value.) The Pure Growth index includes only pure growth stocks; a stock's weight in the index is determined by its growth score; likewise for pure value.

When the index is rebalanced, the relative weights of any two stocks should be proportional to their relative style scores. For instance, in the pure value index, at the rebalancing, a stock with a value score of 1.5 should have 50% more weight than a stock with a value score of 1.0 (since 1.5 is 50% greater than 1.0). Since the weight represents how much of the index's total market value is in each stock, the weight depends on both the price of the stock and the number of shares held. The design of the index calculation procedure determines the number of shares of each stock to be included while the market determines the price.

As in the case of equal weighted indices, the general structure follows that used for the cap-weighted indices, defining the basic equations as:

$$\text{Index Level} = \frac{\text{Index Market Value}}{\text{Divisor}} \quad (22)$$

and

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i \quad (23)$$

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<sup>2</sup> Price weighting means an unweighted or simple average of the stock prices. It is seldom used in new indices but continues to be used by some long-standing indices including the Dow Jones Industrial Average and the Nikkei 225 Stock Average. The Dow dates from 1896; the Nikkei from 1949.

For simplicity, the equations do not include the IWF term even though it is used in the actual calculations. The share count is modified so that the stock's weight reflects its style score rather than its market capitalization. Define  $M\_Shares$  as the modified share count:

$$M\_Shares_i = Shares_i * PWF_i \quad (24)$$

where  $PWF_i$  is the Pure Style Weight Factor for stock  $i$ . The  $PWF_i$  depends on the true share count ( $Shares$ ), the style score and the stock's price value when the index is rebalanced. The  $PWF_i$  is defined as:

$$PWF_i = \frac{k * SV_i}{Shares_{i,t=reb} * Price_{i,t=reb}} \quad (25)$$

$Shares_i$  is the true share count, not the modified share count.  $SV_i$  is the value score of stock  $i$ . The factor  $k$  is included to scale the numbers; the denominator is quite large and without  $k$  the  $PWF_i$  would be a very small number.  $SV$  is the capped value score (to prevent outliers from having high weights in the index, the scores are capped at 2, effectively chopping off a distribution at two standard deviations away from the mean.) The subscript  $t=reb$  indicates that the shares and price for stock  $i$  are evaluated at the rebalancing date. Note also that the IWF would be included in the actual calculation.

By substituting (25) into (24) and then (24) into (23):

$$IndexMarketValue = \sum_i \frac{k * SV_i * P_{i,t} * Shares_{i,t}}{P_{i,t=reb} * Shares_{i,t=reb}} \quad (26)$$

Note that we have substituted for  $M\_shares$  so that the  $Shares$  variable in (26) is the actual shares. Now, re-arranging terms on the right hand side:

$$= k * \sum_i SV_i * \frac{P_{i,t} * Shares_{i,t}}{P_{i,t=reb} * Shares_{i,t=reb}} \quad (27)$$

The fraction term inside the summation sign is one plus the proportional increase or decrease in the value of the stock since the rebalancing (in effect the "value relative"). Therefore, the right hand side of (27) represents the weighted average of the value relatives of the stocks where the weights are the value scores. Recalling (22), we can define the index as:

$$IndexLevel = \frac{k * \sum_i SV_i \frac{P_{i,t} * Shares_{i,t}}{P_{i,t=reb} * Shares_{i,t=reb}}}{Divisor} \quad (28)$$

Users of Standard & Poor's attribute weighted indices typically get the modified index shares and price in daily files from Standard & Poor's, and do not have to do the above calculations themselves. Having a simple share number in terms of the modified index shares helps fit these indices seamlessly into systems designed for market capitalization weighted indices.

### Corporate Actions

<b>Parent Index Action</b>	<b>Adjustment made to Pure Style Index</b>	<b>Divisor adjustment required?</b>
Constituent change	<p>If the constituent being dropped is a member of the Pure Style Index, it is removed from the Pure Style Index.</p> <p>The replacement stock can be added to either the pure value or the pure growth index, or to neither.</p> <p>The weight is simply the ratio of the capped style score of the added stock divided by the sum of style scores of all index constituents. The indicative weight is announced through <i>Standard &amp; Poor's Index Alert</i>.</p>	Yes
Share changes between quarterly share adjustments	<p>The weight of stocks is unchanged.</p> <p>(The share count follows the parent index share count. To keep weights of stocks unchanged following a share change the PWF is adjusted for the stock whose shares are being changed.)</p>	No
Quarterly share changes	<p>If the annual rebalancing date coincides with a quarterly share update, this is the only time when weights are revised.</p>	Yes if it coincides, No otherwise.
Spin-off	<p>The weight of stocks is unchanged.</p> <p>The price follows the parent index price change. To keep weights of stocks unchanged following price change, PWF is adjusted for the stock whose shares are being changed.</p>	No
Rights Offering	<p>The weight of stocks is unchanged.</p> <p>The price follows the parent index price change. To keep weights of stocks unchanged, PWF is adjusted for the stock whose shares are being changed.</p>	No

<b>Parent Index Action</b>	<b>Adjustment made to Pure Style Index</b>	<b>Divisor adjustment required?</b>
Stock Split	Shares are multiplied by and the price is divided by the split factor.	No
Special Dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	Yes

# Leveraged and Inverse Indices

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## Leveraged Indices

Standard & Poor's Leveraged Indices are designed to generate a multiple of the return of the underlying index in situations where the investor borrows funds to generate index exposure beyond his/her cash position. The approach is to first calculate the underlying index, then calculate the daily returns for the leveraged index and, finally, to calculate the current value of the leveraged index by incrementing the previous value by the daily return. There is no change to the calculation of the underlying index.

The daily return for the leveraged index consists of two components: (1) the return on the total position in the underlying index less (2) the borrowing costs for the leverage.

The formula for calculating the Leveraged Index is as follows:

$$\text{Leveraged Index Return} = K * \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - (K - 1) * \left( \frac{\text{Borrowing Rate}}{360} \right) * D_{t,t-1} \quad (29)$$

In equation (29) the borrowing rate is applied to the leveraged index value because this represents the funds being borrowed. Given this, the Leveraged Index Value at time  $t$  can be calculated as:

$$\text{Leveraged Index Value}_t = (\text{Leveraged Index Value}_{t-1}) * (1 + \text{Leveraged Index Return}) \quad (30)$$

Substituting (29) into (30) and expanding the right hand side of (30) yields:

$$\text{Leveraged Index Value}_t = \text{Leveraged Index Value}_{t-1} * \left[ 1 + \left[ K * \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - (K - 1) * \left[ \frac{\text{Borrowing Rate}}{360} \right] * D_{t,t-1} \right] \right] \quad (31)$$

where:

$K$  ( $K \geq 1$ ) = Leverage Ratio

- $K = 1$ , no leverage
- $K = 2$ , Exposure = 200%
- $K = 3$ , Exposure = 300%

*Borrowing Rate* = Overnight LIBOR in the U.S. or EONIA in Europe

$D_{t,t-1}$  = the number of calendar days between date  $t$  and  $t-1$

In the absence of leverage ( $K=1$ ),

$$\text{Leveraged Index Value}_t = \text{Leveraged Index Value}_{t-1} \left[ \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right] \quad (32)$$

The leverage position is rebalanced daily. This is consistent with the payoff from futures based replication.

### Inverse Indices

Standard & Poor's Inverse indices are designed to provide the inverse performance of the underlying index; this represents a short position in the underlying index. When an investor holds a short position he/she must pay dividends and interest for the borrowed stock. The calculation follows the same general approach as the leveraged index with certain adjustments: First, the return on the underlying index is reversed and is based on the total return of the underlying index so that dividends and price movements are included. Second, while the costs of borrowing the securities are not included, there is an adjustment to reflect the interest earned on both the initial investment and the proceeds from selling short the securities in the underlying index. These assumptions reflect normal industry practice. (Straightforward adjustments can be made to either to include the costs of borrowing securities or to exclude the interest earned on the shorting proceeds and the initial investment.)

The general formula for the return to the inverse index is

$$\begin{aligned} \text{Inverse Index Return} = & -K * \left( \frac{\text{Underlying Index Total Return}_t}{\text{Underlying Index Total Return}_{t-1}} - 1 \right) \\ & + (K + 1) * \left( \frac{\text{Lending Rate}}{360} \right) D_{t,t-1} \end{aligned} \quad (33)$$

Where the first right hand side term represents the total return on the underlying index and the second right hand side term represents the interest earned on the initial investment and the shorting proceeds.

Expanding this as done above for the leveraged index yields:

$$\begin{aligned} \text{Inverse Index Value}_t = & \text{Inverse Index Value}_{t-1} * \left[ 1 - \left[ K * \left( \frac{\text{Underlying Index TR}_t}{\text{Underlying Index TR}_{t-1}} - 1 \right) - (K + 1) * \left[ \frac{\text{Lending Rate}}{360} \right] * D_{t,t-1} \right] \right] \end{aligned} \quad (34)$$

where:

$K (K \geq 1)$  = Leverage Ratio

- $K = 1$ , Exposure = -100%
- $K = 2$ , Exposure = -200%
- $K = 3$ , Exposure = -300%

*Lending Rate* = Overnight LIBOR in the U.S. or EONIA in Europe

$D_{t,t-1}$  = the number of calendar days between date  $t$  and  $t-1$

In the absence of leverage ( $K = 1$ ),

$$\begin{aligned} & \text{InverseIndexValue}_t = \\ & \text{Inverse IndexValue}_{t-1} * \left[ 1 - \left[ \left( \frac{\text{UnderlyingIndexTR}_t}{\text{UnderlyingIndexTR}_{t-1}} - 1 \right) - (2) * \left[ \frac{\text{LendingRate}}{360} \right] * D_{t,t-1} \right] \right] \end{aligned} \quad (35)$$

The inverse position is rebalanced daily. This is consistent with the payoff from futures based replication.

# Total Return Calculations

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The preceding discussions were related to price indices where changes in the index level reflect changes in stock prices. In a total return index changes in the index level reflect both movements in stock prices and the reinvestment of dividend income. A total return index represents the total return earned in a portfolio that tracks the underlying price index and reinvests dividend income in the overall index, not in the specific stock paying the dividend.

The total return construction differs from the price index and builds the index from the price index and daily total dividend returns. The first step is to calculate the total dividend paid on a given day and convert this figure into points of the price index:

$$TotalDailyDividend = \sum_i Dividend_i * Shares_i \quad (36)$$

Where *Dividend* is the dividend per share paid for stock *i* and *Shares* are the shares. This is done for each trading day. *Dividend<sub>i</sub>* is generally zero except for four times a year when it goes ex-dividend for the quarterly dividend payment. Some stocks do not pay a dividend and *Dividend* is always zero. *TotalDailyDividend* is measured in dollars. This is converted to index points by dividing by the divisor for the underlying price index:

$$IndexDividend = \frac{TotalDailyDividend}{Divisor} \quad (37)$$

The next step is to apply the usual definition of a total return from a financial instrument to the price index. Equation (38) gives the definition, equation (39) applies it to the index:

$$Total\ Return = \left( \frac{P_t + D_t}{P_{t-1}} \right) - 1 \quad (38)$$

and

$$DTR_t = \left( \frac{IndexLevel_t + IndexDividend_t}{IndexLevel_{t-1}} - 1 \right) \quad (39)$$

where the *TotalReturn* and the daily total return for the index (*DTR*) is stated as a decimal. The *DTR* is used to update the total return index from one day to the next:

$$Total\ Return\ Index_t = (Total\ Return\ Index_{t-1}) * (1 + DTR_t) \quad (40)$$

# Index Fundamentals

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Indices are often used to measure market conditions or gauge valuations among markets or between stocks and indices through measures like earnings per share (EPS), price-earnings ratios, dividend yields and so forth. These are calculated by using the divisor as if it represents shares for a company. The basic format is illustrated for the EPS for an index:

$$Index\ EPS = \frac{\sum_i eps_i * shares_i}{Divisor} \quad (41)$$

where *IndexEPS* is the EPS for the overall index, *eps<sub>i</sub>* is the EPS for stock *i* and *shares<sub>i</sub>* are the shares used to calculate the index with any adjustments such as the IWF incorporated into the figure. If the calculation refers to an equal weighted or attribute weighted index, the calculation use the shares defined for those indices (C\_shares or M\_shares, as appropriate).

The price-earnings (PE) ratio for the index is simply the ratio of the index level (or price) to the index EPS. For a cap-weighted index, this can also be calculated directly from the stock level data by dividing the total market cap of the index by total earnings of all companies in the index. In this calculation, the Divisor terms in the denominator drop out:

$$IndexPE = \frac{\frac{\sum P_i * Shares_i}{Divisor}}{\frac{\sum eps_i * Shares_i}{Divisor}} \quad (42)$$

The same general approach can be used for various index fundamentals and ratios such as book value per share, price-to-book, dividend-to-price (i.e. dividend yield) and so forth.

## Fund Strategy Indices

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A fund strategy index is a portfolio of funds that is designed to deliver excess returns over a benchmark within an established risk framework. A fund strategy index can include a portfolio of funds for a single asset class (i.e. U.S. equities) or cover multiple asset classes (i.e. asset allocation programs). Fund strategy indices are created with U.S. mutual funds for U.S. investors or International (offshore) funds for non-U.S. investors.

Each fund's weight within the strategy index portfolio is determined by a rules-based portfolio construction methodology. The calculation process for fund strategy indices are based upon the divisor method commonly used for S&P equity indices. However, the share count is redefined to be a number that will achieve the fund's allocation weighting at the rebalancing date.

$$\text{Index Level} = \frac{\text{Index Market Value}}{\text{Divisor}} \quad (43)$$

and

$$\text{Index Market Value} = \sum_i \text{NAV}_i * \text{Shares}_i \quad (44)$$

where NAV is the Net Asset Value, a fund's share price.

To calculate a fund index, shares are redefined. Rather than being the actual count of shares multiplied by the investable weight factors (IWF) or other such adjustment factors, the *Shares* number is calculated to establish the allocation weighting at the rebalancing date. For clarity, this section will refer to the "shares" figure as *fund\_shares*. Not only are these not the true share count, they have essentially no relation to a fund's true share count. Since *fund\_shares* are being used instead of true shares, the Index Market Value defined in (44) is not the actual market value of the index. In a fund strategy index the Index Market Value is an arbitrary or nominal value for the portfolio used when the *fund\_shares* figure is established. The *fund\_shares* are calculated at each rebalancing:

$$\text{fund\_shares}_{i, \text{rebalancing date}} = \frac{w_{i, \text{rebalancing date}}}{\text{NAV}_{i, \text{rebalancing date}}} \quad (45)$$

where  $w_i$  is the new allocation weight and  $\text{NAV}_i$  is the price of the *fund<sub>i</sub>* at the rebalancing date.

At each index rebalancing, in order to maintain index series continuity, it is also necessary to adjust the divisor.

$$(Index\ Level)_{before\ rebalancing} = (Index\ Level)_{after\ rebalancing} \quad (46)$$

Therefore,

$$(Divisor)_{after\ rebalancing} = \frac{(Index\ Market\ Value)_{after\ rebalancing}}{(Index\ Level)_{before\ rebalancing}} \quad (47)$$

where,

$$Index\ Market\ Value = \sum_i NAV_i * fund\_shares_i$$

### Total Return and Synthetic Price Indices

Some funds, primarily international (offshore), provide investors with accumulating or distributing share classes. The accumulating share class does not declare nor pay out net income and/or net realized capital gains. Net income and/or net realized capital gains are retained and included in the net asset value per share. The distributing share class declares and pays out net income and net capital gains. For consistency, S&P publishes total return and synthetic price return indices.

### Total Return Index

The total return index assumes dividend and capital gain distributions are reinvested in the index. Note that the results will be different if an investor held all funds in an index in the correct proportions and instructed each fund company to reinvest distributions into the fund paying those distributions. On any given date  $t$ :

$$Total\ Return\ Index_t =$$

$$Total\ Return\ Index_{t-1} * Total\ Return\ Multiplier_t$$

where

$$Total\ Return\ Multiplier_t =$$

$$\frac{Index\ Level_t + Index\ Distribution\ Points_t}{Index\ Value_{t-1}} \quad (48)$$

and,

$$Index\ Distribution\ Points_t =$$

$$\frac{\sum_{i=1}^N fund\_shares_{i,t} * Distributions_{i,t}}{Divisor_t} \quad (49)$$

### Synthetic Price Index

The synthetic price index is calculated from the total return strategy index and is based upon the ratio of the benchmark's total return index to its price index from the base date of the fund strategy index.

Price Index<sub>t</sub> =

$$\frac{Price\ Index_{base\ date} * \left( \frac{Total\ Return\ Index_t}{Total\ Return\ Index_{base\ date}} \right)}{\left( \frac{Benchmark\ Price\ Index_{base\ date}}{Benchmark\ Price\ Index_t} \right) * \left( \frac{Benchmark\ TR\ Index_t}{Benchmark\ TR\ Index_{base\ date}} \right)} \quad (50)$$

### Index and Corporate Actions

S&P fund index action	Adjustment made to fund index	Divisor adjustment for the fund index?
Constituent Change	The fund entering the index goes in at the weight of the fund coming out. This weight is used to compute fund_shares of the added fund, using Equation 45.	No.
Fund added/deleted	Net change in fund index market value determines divisor adjustment.	Yes.

Corporate Action	Adjustment made to strategy index	Divisor adjustment for strategy index
Fund Closing	Funds must be open to new investments. Fund will be removed from the index.	Yes, if fund removed without replacement. No, if fund replaced.
Fund Split	Fund_shares are multiplied by and NAV is divided by the split factor.	No.
Fund Merger – fund is the acquiring fund.	Assets will be added to the fund at NAV. No change in fund_shares.	No.
Fund Merger – fund is merged into another fund.	Fund no longer available. Fund will be removed from the index.	Yes, if fund removed without replacement. No, if fund replaced.
Fund Liquidation	Fund no longer available. Fund will be removed from the index.	Yes, if fund removed without replacement. No, if fund replaced.

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