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Revised Insurance Risk-Based Capital Model Charge Methodology For Common Equities

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Standard & Poor's Ratings Services is updating the way it assesses the capital adequacy of insurance companies worldwide. This new risk-based capital model is described in the RatingsDirect article "Request For Comment: Revisions In The Risk-Based Insurance Capital Model". In the model, the capital charges applied to unaffiliated common stock holdings reflect observed market volatility patterns at various confidence levels. A regime-switching lognormal model was selected to generate the modeled equity capital charge for each target rating. This model was chosen to address the shortcomings of the independent lognormal model for equity returns. The regime-switching model yields a more realistic approach, one more closely aligned with prevailing notions about the distribution of equity returns.

A common assumption in equity price models is that equity prices follow a geometric Brownian motion. This is equivalent to the assumption that price changes follow the lognormal distribution and continuously compounded returns follow the normal distribution. In addition, constant volatility is a common assumption. The lognormal model, both mathematically tractable and intuitive, underlies many financial models, including the Black-Scholes formula. However, empirical evidence strongly suggests that observed equity returns in fact violate the lognormal assumption. Historical returns data exhibits fat tails (excess kurtosis) and a negative skew. These qualities are well understood by market practitioners and are widely reflected in the literature. The regime-switching model not only incorporates the fat tails and negative skew implied by the historical data, but also allows for non-constancy of volatility, providing a closer fit to observed returns.

The regime-switching model reflects the assumption that the equities can switch between periods with differing volatilities. For example, a stable period characterized by a relatively higher mean return and relatively lower volatility can be followed by a less stable period characterized by a relatively lower mean return and relatively higher volatility. This switching mechanism allows for stochastic volatility, replacing the constant volatility assumption of the lognormal model. In deriving the equity capital charge, Standard & Poor's uses two distinct periods, or "regimes". Within each regime, returns are assumed to follow a lognormal distribution with regime-specific parameters. In addition, given that the process is in either regime, there exists an associated probability of transitioning between regimes. The transition process is assumed to be Markovian, in that the probability of transition depends only upon the current state, and not upon previous states. The process randomly switches between the two lognormal processes, with the probabilities of switching regimes given by the estimated transition probabilities. This process not only produces the desired fatter tails, but also captures stochastic volatility in a simple, yet effective, manner.

For each country under consideration, the model was parameterized to 30 years of monthly returns data from the respective MSCI index for the country. The parameters estimated were the mean return for each regime, the volatility for each regime, and the two transition probabilities of switching between regimes. The parameters were estimated using maximum likelihood estimation. Ten thousand monthly equity returns paths were then simulated, making use of the estimated parameters and a high-quality random number generator. For each target rating, confidence levels associated with five-year cumulative default probabilities were obtained from Standard & Poor's default studies. These confidence levels were mapped to percentiles of the one-year returns distribution of the 10,000

simulated paths to produce the capital charges. The average of the resulting charges from four 10,000-path simulations was calculated and constituted the final charge.

Technical Notes

Technical Note

Geometric Brownian Motion

The process within each regime is modeled as geometric Brownian motion, and evolves according to the following stochastic differential equation:

$$dS(t) = \mu S(t)dt + \sigma S(t) dZ(t)$$

where

- S(t): stock price at time t
- dS(t): instantaneous change in S(t) during time increment dt
- μ : instantaneous mean drift rate
- dt: instantaneous change in time
- σ : instantaneous volatility of S(t)
- Z(t): Weiner process, distributed $N(0, \sqrt{dt})$
- dZ(t): instantaneous change in Weiner process during increment dt

In the simulation, the discrete form of geometric Brownian motion is used. This takes the following form:

$$\Delta S(t) = \mu S(t)\Delta t + \sigma S(t) \varepsilon \sqrt{\Delta t}$$

where

- ε : a draw from a standard normal distribution ($N(0,1)$)
- Δt : a small time increment
- $\Delta S(t)$ the change in S(t) during the increment Δt

Lognormal Distribution

The lognormal assumption implies that equity prices follow the lognormal distribution and continuously compounded returns follow the normal distribution. Thus, for $t > r$, $\ln(S(t)/S(r))$ follows the normal distribution, and both $S(t)/S(r)$ and $S(t)$ follow the lognormal distribution. Within, each regime, the lognormal distribution for equity prices is assumed.

Parameter Estimation*

Maximum likelihood estimation was used to estimate the six parameters of the model. The likelihood function is

$$L(\Theta) = f(Y_1|\Theta)f(Y_2|\Theta, y_1)f(Y_3|\Theta, y_1, y_2)(fY_4|y_1, y_2, y_3) \dots f(Y_n|\Theta, y_1, y_2, y_3, \dots, y_{n-1})$$

where Y_t is the continuously compounded return for the $t+1^{th}$ month, Θ is the parameter set, and the y_i 's are the previously realized returns (history set).

We maximize this likelihood function by calculating recursively, for each month t

$$f(p_t, p_{t-1}, y_t | y_{t-1}, \dots, y_1, \Theta) = p(p_{t-1} | y_{t-1}, \dots, y_1, \Theta) * p(p_t | p_{t-1}, \Theta) f(y_t | p_{t-1}, \Theta)$$

where the p_i 's represent the regime the process is in at time t . The term $p(p_{t-1} | y_{t-1}, \dots, y_1, \Theta)$ (the conditional probability of the previous regime state) is calculated recursively; the initial distribution of p_1 makes use of π_1 and π_2 , the estimated unconditional probabilities that the process is in regime 1 or regime 2. π_1 and π_2 are estimated by the equations

$$\begin{aligned} \pi_1 &= p_{21} / (p_{21} + p_{12}) \\ \pi_2 &= 1 - \pi_1 \end{aligned}$$

where p_{ij} represents the probability that the process will transition from state i to state j in any given time step. The parameter set Θ is made up of six parameters, $\mu_1, \mu_2, \sigma_1, \sigma_2, p_{12}$, and p_{21} . The parameters are estimated using maximum likelihood estimation. The parameters were estimated for each country's MSCI index.

* Regime-switching technique and parameter estimation procedure adapted from Mary R. Hardy's "A Regime Switching Model of Long-Term Stock Returns," *North American Actuarial Journal*. 5(2), 41-53.

Simulation Technique

The simulation technique involved the generation of monthly returns paths. For each path, the initial regime was selected using the unconditional probabilities π_1 and π_2 . Once the initial regime was chosen, the algorithm simulated monthly returns by randomly drawing from the regime-specific estimated distribution. After the return value for the month was drawn, the algorithm compared a random draw from the uniform distribution with the appropriate transition probability to select the regime assumed for the following month. The model simulated 10,000 such paths. The return paths were then transformed into price paths. The quantiles from the one-year simulated returns distribution were mapped to the target rating-specific confidence level to arrive at the equity charge. The final charge applied for each country was the average of the calculated charges from four 10,000-path simulations. Analyst adjustments were applied to the final charges where appropriate.

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